Asymptotics of first-passage percolation on 1-dimensional graphs

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(Submitted on 12 Jul 2011)

In this paper we consider standard first-passage percolation on certain 1-dimensional periodic graphs. One such graph of particular interest is the $\Z\times\{0,1,...,K-1\}^{d-1}$ nearest neighbour graph for $d,K\geq1$. Let $T(u,v)$ denote the time it takes for an infection started at u to reach v, and let $N(u,v)$ denote the length of the geodesic (path with minimal passage time) from u to v. We derive asymptotic results that show how the behaviour of first-passage percolation on 1-dimensional graphs differ from what is known or expected in higher dimensions. Let $\mathbf{n}=(n,0,...,0)$. By subadditivity $T(0,\mathbf{n})/n\raw\mu$ for some $\mu>0$ as $n\raw\infty$, almost surely and in L^1. We show that for some $\sigma>0$, as $n\raw\infty$, $\big(T(0,\mathbf{n})-\mu n\big)/\sigma\sqrt{n}$ converges in distribution to a standard normal, and moreover, that $\limsup_{n\raw\infty}\big(T(0,\mathbf{n})-\mu n\big)/\sigma\sqrt{2n\log\log n}=1$, almost surely. We further prove that $\E\big[T(0,\mathbf{n})\big]$ and $\Var\big(T(0,\mathbf{n})\big)$ are monotonic in n, for large enough n. Results for $N(0,\mathbf{n})$ corresponding to the results mentioned for $T(0,\mathbf{n})$ are also derived.

We also allow different sets of initially infected vertices, and construct an exact coupling of two infections with different starting configurations. Using this coupling we prove a 0--1 law.

Subjects:	**Probability (math.PR)**
MSC classes:	60K35, 60F05, 60F15, 60F17, 60F20
Cite as:	**arXiv:1107.2276 [math.PR]**
	(or **arXiv:1107.2276v1 [math.PR]** for this version)

Submission history

From: Daniel Ahlberg [view email]

[v1] Tue, 12 Jul 2011 13:14:07 GMT (62kb)

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