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Randomly Stopped Nonlinear Fractional Birth Processes

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We present and analyse the nonlinear classical pure birth process $\mathpzc {N} (t)$, $t>0$, and the fractional pure birth process $\mathpzc{N}^\nu (t)$, $t>0$, subordinated to various random times, namely the first-passage time T_t of the standard Brownian motion $B(t)$, $t>0$, the α-stable subordinator $\mathpzc{S}^\alpha(t)$, $\alpha \in (0,1)$, and others. For all of them we derive the state probability distribution $\hat{p}_k (t)$, $k \geq 1$ and, in some cases, we also present the corresponding governing differential equation. We also highlight interesting interpretations for both the subordinated classical birth process $\hat{\mathpzc{N}} (t)$, $t>0$, and its fractional counterpart $\hat{\mathpzc{N}}^\nu (t)$, $t>0$ in terms of classical birth processes with random rates evaluated on a stretched or squashed time scale. Various types of compositions of the fractional pure birth process $\mathpzc{N}^\nu(t)$ have been examined in the last part of the paper. In particular, the processes $\mathpzc{N}^\nu(T_t)$, $\mathpzc{N}^\nu(\mathpzc {S}^\alpha(t))$, $\mathpzc{N}^\nu(T_{2\nu}(t))$, have been analysed, where $T_{2\nu}(t)$, $t>0$, is a process related to fractional diffusion equations. Also the related process $\mathpzc{N}(\mathpzc{S}^\alpha({T_{2\nu}(t)}))$ is investigated and compared with $\mathpzc{N}(T_{2\nu}(\mathpzc{S}^\alpha(t))) = \mathpzc{N}^\nu (\mathpzc{S}^\alpha(t))$. As a byproduct of our analysis, some formulae relating Mittag--Leffler functions are obtained.

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