Mathematics > Probability

A trajectorial interpretation of the dissipations of entropy and Fisher information for stochastic differential equations

Joaquin Fontbona, Benjamin Jourdain (CERMICS)

(Submitted on 17 Jul 2011)

We introduce and develop a pathwise description of the dissipation of general convex entropies for continuous time Markov processes, based on simple backward martingales and convergence theorems with respect to the tail sigma field. The entropy is in this setting the expected value of a backward submartingale. In the case of (non necessarily reversible) Markov diffusion processes, we use Girsanov theory to explicit its Doob-Meyer decomposi- tion, thereby providing a stochastic analogue of the well known entropy dissipation formula, valid for general convex entropies (including total variation). Under additional regularity assumptions, and using It\^o calculus and ideas of Arnold, Carlen and Ju [2] we obtain a new Bakry Emery criterion which ensures exponential convergence of the entropy to 0. This criterion is non-intrisic since it depends on the square root of the diffusion matrix, and cannot be written only in terms of the diffusion matrix itself. Last, we provide an example where the classic Bakry Emery criterion fails, but our non-intrisic criterion ensuring exponential convergence to equilibrium applies without modifying the law of the diffusion process.

Subjects: **Probability (math.PR)**

Cite as: **arXiv:1107.3300v1 [math.PR]**

Submission history

From: Joaquin Fontbona [view email]

[v1] Sun, 17 Jul 2011 13:07:18 GMT (36kb)