Mathematics > Probability

A Representation Theorem for Smooth Brownian Martingales

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We show that, under certain smoothness conditions, a Brownian martingale at a fixed time can be represented as an exponential of its value at a later time. The time-dependent generator of this exponential operator is equal to one half times the Malliavin derivative. This result can also be seen as a generalization of the semi-group theory of parabolic partial differential equations to the parabolic path-dependent partial differential equations introduced by Dupire (2009) and Cont and Founi\'e (2011). The exponential operator can be calculated explicitly in a series expansion, which resembles the Dyson series of quantum mechanics. Our continuous-time martingale representation result is proved by a passage to the limit of a special case of a backward Taylor expansion of an approximating discrete-time martingale. The latter expansion can also be used for numerical calculations.

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