Exact Solutions to the Six-Vertex Model with Domain Wall Boundary Conditions and Uniform Asymptotics of Discrete Orthogonal Polynomials on an Infinite Lattice

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Liechty, Karl Edmund

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Committee Members:	Its, Alexander R. Lempert, Lazlo Kitchens, Bruce, 1953-
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Abstract:

In this dissertation the partition function, Z_n, for the six-vertex model with domain wall boundary conditions is solved in the thermodynamic limit in various regions of the phase diagram. In the ferroelectric phase region, we show that $Z_n=CG^nF^{n^2}(1+O(e^{-n^{1-\ep}}))$ for any $\ep>0$, and we give explicit formulae for the numbers C, G, and F. On the critical line separating the ferroelectric and disordered phase regions, we show that $Z_n=Cn^{1/4}G^{\sqrt{n}}F^{n^2}(1+O(n^{-1/2}))$, and we give explicit formulae for the numbers G and F. In this

phase region, the value of the constant C is unknown. In the antiferroelectric phase region, we show that $Z_n=C\th_4(n\om)F^{n^2}(1+O(n^{-1}))$, where $\th_4$ is Jacobi's theta function, and explicit formulae are given for the numbers $\om$ and F. The value of the constant C is unknown in this phase region. In each case, the proof is based on reformulating Z_n as the eigenvalue partition function for a random matrix ensemble (as observed by Paul Zinn-Justin), and evaluation of large n asymptotics for a corresponding system of orthogonal polynomials. To deal with this problem in the antiferroelectric phase region, we consequently develop an asymptotic analysis, based on a Riemann-Hilbert approach, for orthogonal polynomials on an infinite regular lattice with respect to varying exponential weights. The general method and results of this analysis are given in Chapter 5 of this dissertation.

Description:

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