

Exact Solutions to the Six-Vertex Model with Domain Wall Boundary Conditions and Uniform Asymptotics of Discrete Orthogonal Polynomials on an Infinite Lattice

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Exact Solutions to the Six-Vertex Model with Domain Wall Boundary Conditions and Uniform Asymptotics of Discrete Orthogonal Polynomials on an Infinite Lattice

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Abstract:

In this dissertation the partition function, Z_n , for the six-vertex model with domain wall boundary conditions is solved in the thermodynamic limit in various regions of the phase diagram. In the ferroelectric phase region, we show that $Z_n = C G^n F^{n^2} (1 + O(e^{-n^{1-\epsilon}}))$ for any $\epsilon > 0$, and we give explicit formulae for the numbers C , G , and F . On the critical line separating the ferroelectric and disordered phase regions, we show that $Z_n = C n^{1/4} G^{\sqrt{n}} F^{n^2} (1 + O(n^{-1/2}))$, and we give explicit formulae for the numbers G and F . In this

phase region, the value of the constant C is unknown. In the antiferroelectric phase region, we show that $Z_n = C \theta_4(n) F^{n^2} (1 + O(n^{-1}))$, where θ_4 is Jacobi's theta function, and explicit formulae are given for the numbers C and F . The value of the constant C is unknown in this phase region. In each case, the proof is based on reformulating Z_n as the eigenvalue partition function for a random matrix ensemble (as observed by Paul Zinn-Justin), and evaluation of large n asymptotics for a corresponding system of orthogonal polynomials. To deal with this problem in the antiferroelectric phase region, we consequently develop an asymptotic analysis, based on a Riemann-Hilbert approach, for orthogonal polynomials on an infinite regular lattice with respect to varying exponential weights. The general method and results of this analysis are given in Chapter 5 of this dissertation.

Description:

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