



On a Devil's staircase associated to the joint spectral radii of a family of pairs of matrices

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The joint spectral radius of a finite set of real $d \times d$ matrices is defined to be the maximum possible exponential rate of growth of products of matrices drawn from that set. In previous work with K. G. Hare and J. Theys we showed that for a certain one-parameter family of pairs of matrices, this maximum possible rate of growth is attained along Sturmian sequences with a certain characteristic ratio which depends continuously upon the parameter. In this paper we answer some open questions from that paper by showing that the dependence of the ratio function upon the parameter takes the form of a Devil's staircase. We show in particular that this Devil's staircase attains every rational value strictly between 0 and 1 on some interval, and attains irrational values only in a set of Hausdorff dimension zero. This result generalises to include certain one-parameter families considered by other authors. We also give explicit formulas for the preimages of both rational and irrational numbers under the ratio function, thereby establishing a large family of pairs of matrices for which the joint spectral radius may be calculated exactly.

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