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Posets of Geometric Graphs

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A geometric graph G(bar) is a simple graph drawn in the plane, on points in general position, with straight-line edges. We call G(bar) a geometric realization of the underlying abstract graph G. A geometric homomorphism is a vertex map that preserves adjacencies and crossings (but not necessarily non-adjacencies or non-crossings). This work uses geometric homomorphisms to introduce a partial order on the set of isomorphism classes of geometric realizations of an abstract graph G. We say G(bar) precedes G (hat) if G(bar) and G(hat) are geometric realizations of G and there is a vertex-injective geometric homomorphism from G(bar) to G(hat). This paper develops tools to determine when two geometric realizations are comparable. Further, for 3 \leq n \leq 6, this paper provides the isomorphism classes of geometric realizations of P_n, C_n and K_n, as well as the Hasse diagrams of the geometric homomorphism posets of these graphs. The paper also provides the following results for general n: the poset of P_n and C_n has a unique minimal element and a unique maximal element; if k \leq n then the poset of P_k (resp., the poset of C_k) is a subposet of the poset for P_n (resp., C_n); and the poset for K_n contains a chain of length n-2.

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