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On the enumeration of three-rowed standard Young tableaux of skew shape in terms of Motzkin numbers

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The enumeration of standard Young tableaux (SYTs) of shape {\lambda} can be easily computed by the hook-length formula. In 1981, Amitai Regev proved that the number of SYTs having at most three rows with n entries equals the nth Motzkin number M_n. In 2006, Regev conjectured that the total number of SYTs of skew shape {\lambda}/(2, 1) over all partitions {\lambda} having at most three parts with n entries is the difference of two Motzkin numbers, M_{n-1} - M_{n-3}. Ekhad and Zeilberger proved Regev's conjecture using a computer program. In 2009, S.-P. Eu found a bijection between Motzkin paths and SYTs of skew shape with at most three rows to prove Regev's conjecture, and Eu also indirectly showed that for the fixed {\mu} = ({\mu}1,{\mu}2) the number of SYTs of skew shape {\lambda}/{\mu} over all partitions {\lambda} having at most three parts can be expressed as a linear combination of the Motzkin numbers. In this paper, we will find an explicit formula for the generating function for the general case: for each partition {\mu} having at most three parts the generating function gives a formula for the coefficients of the linear combination of Motzkin numbers. We will also show that these generating functions are unexpectedly related to the Chebyshev polynomials of the second kind.

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