Constructing graphs with no immersion of large complete graphs

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In 1989, Lescure and Meyniel proved, for $d=5, 6$, that every d-chromatic graph contains an immersion of K_d, and in 2003 Abu-Khzam and Langston conjectured that this holds for all d. In 2010, DeVos, Kawarabayashi, Mohar, and Okamura proved this conjecture for $d = 7$. In each proof, the d-chromatic assumption was not fully utilized, as the proofs only use the fact that a d-critical graph has minimum degree at least $d - 1 $. DeVos, Dvo\v{r}\'ak, Fox, McDonald, Mohar, and Scheide show the stronger conjecture that a graph with minimum degree $d-1$ has an immersion of K_d fails for $d=10$ and $d\geq 12$ with a finite number of examples for each value of d, and small chromatic number relative to d, but it is shown that a minimum degree of $200d$ does guarantee an immersion of K_d.
In this paper we show that the stronger conjecture is false for $d=8,9,11$ and give infinite families of examples with minimum degree $d-1$ and chromatic number $d-3$ or $d-2$ that do not contain an immersion of K_d. Our examples can be up to $(d-2)$-edge-connected. We show, using Haj\'os' Construction, that there is an infinite class of non-$(d-1)$-colorable graphs that contain an immersion of K_d. We conclude with some open questions, and the conjecture that a graph G with minimum degree $d - 1$ and more than $\frac{|V(G)|}{1+m(d+1)}$ vertices of degree at least md has an immersion of K_d.

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