关于累积和(CUSUM)检验的改进

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摘要　对连续检验问题,常用的检测方法有三大类,其一是众所周知的Shewhart控制图,它是最常用的对生产过程进行连续监控的控制方法,不过,如果过程均值有小的漂移(即$\mu-\mu\_0$小)时,Shewhart控制图的检验效果不是很好,除了Shewhart控制图外,另有二类常用的控制图法,其一是累积和控制图(CUSUM),由page~[1]基于似然比导出, 其二是指数加权移动平均控制图（EWMA）, 由Roberts~[2]给出,它们已被证明在检验小的漂移时效过不错,许多人对CUSUM与EWMA进行了比较 , 总的来说,最好的CUSUM与最好的EWMA在检验小的漂移方面难分优劣,但CUSUM是由似然比导出的,且其平均运行长度的计算相对来说要简便一些,因此, CUSUM比EWMA更具优势,应用更广。我们分析了CUSUM的导出过程和公式,指出CUSUM有二个可以进一步改进的地方。在此基础上, 我们给出了二个新的累积和检验统计量及其判断准则,它们分别是PCUSUM检验统计量P_n和DOCUSUM检验统计量S_n。在连续检验问题种判断一个检验方法好坏的最重要的标准是平均运行长度,比较标准是：自在要求具有相同的受控状态下平均运行长度ARL_o的条件下，比较起失控状态下的平均运行长度ARL_1,ARL_1越小越好。我们对PCUSUM和DCUSUM检验都建了了其平均运行长度ARL的计算公式。通过对CUSUM，PCUSUM，DCUDUM的平均运行长度的比较,我们发现,我们提出的新的累积和控制方法却比原来的CUSUM有较大改进。

关键词　连续检验问题,累积和控制图,平均运行长度

分类号

ON THE IMPROVING OF CUMULATIVE SUM CHART

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Abstract In general, there are three methods to deal with the continuous inspection problem. One is the well-known Shewhart chart proposed by [3]?Shewhart (1924). Many authors pointed out that when the shift in mean is small (i.e. n ?u0 is small) then the Shewhart chart is mot effective.the other two methods are the CUSUM(Cumulative Sum) chart proposed by Page~[1] and the EWMA (exponential weighted movving average)chart proposed by Roberts~[2].Lots of authors compared the CUSUM with EWMA.Generally speaking,the optimal CUSUM and the optimal EWMA both act well in detecting the small shift in mean.Since the CUSUM is most comparable in practice. The CUSUM text statistic was proposed by Page~[1] basedon the likelihood ratio.Wepointed out that we can inprone the CUSUM intwo ways and then we ginetwo new CUSUM test statistics and their decision principle.One is the P_n named by PCUSUM,and the other is the S_n named by DCUSUM. To judge a inspection method,one ususlly compute its ARL(average run lengeh).A good test should has the smallest out-of-control ARL with the same in-control ARL.We established the formula to compute the ARL of the PCUSUM and the DCUSUM。The results of ARL show that the PCUSUM and the DCUSUM both impromved the ususal CUSUM.

Key words　Continuous inspection problem　cumulative sum charts　average run length

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