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Original Articles

Exponential Stability of Traveling Pulse Solutions of a Singularly Perturbed System ofIntegral Differential Equations Arising From Excitatory Neuronal Networks

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摘要 We establish the exponential stability of fast traveling pulse solutions to nonlinear singularly perturbed systems of integral differential equations arising from neuronal networks. It has been proved that exponential stability of these orbits is equivalent to linear stability. Let $\LL$ be the linear differential operator obtained by linearizing the nonlinear system about its fast pulse, and let $\s(\LL)$ be the spectrum of $\LL$. The linearized stability criterion says that if max\{Re$\l$: $\l\in\s(\LL)$, $\l\neq0\}\x-D$, for some positive constant D, and $\l=0$ is a simple eigenvalue of $\LL(\e)$, then the stability follows immediately (see [13] and [37]). Therefore, to establish the exponential stability of the fast pulse, it suffices to investigate the spectrum of the operator $\LL$. It is relatively easy to find the continuous spectrum, but it is very difficult to find the isolated spectrum. The real part of the continuous spectrum has a uniformly negative upper bound, hence it causes no threat to the stability. It remains to see if the isolated spectrum is safe.

\newline \ \ \ \ Eigenvalue functions (see [14] and [35,36]) have been a powerful tool to study the isolated spectrum of the associated linear differential operators because the zeros of the eigenvalue functions coincide with the eigenvalues of the operators. There have been some known methods to define eigenvalue functions for nonlinear systems of reaction diffusion equations and for nonlinear dispersive wave equations. But for integral differential equations, we have to use different ideas to

construct eigenvalue functions. We will use the method of

variation of parameters to construct the eigenvalue functions in

the complex plane $\C$. By analyzing the eigenvalue functions, we

find that there are no nonzero eigenvalues of $\LL$ in

$\{\l\in\C$: Re$\l\y-D\}$ for the fast traveling pulse. Moreover

$\l=0$ is simple. This implies that the exponential stability of

the fast orbits is true.
关键词 Integral differential equations, traveling pulse solutions, exponential stability, linear differential operators, eigenvalue problems, eigenvalue functions

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Abstract

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