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Original Articles

A Relaxation Scheme for Solving the Boltzmann Equation Based on the Chapman-Enskog Expansion

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摘要 In [16] a visco-elastic relaxation system, called the relaxed Burnett system, was proposed by Jin and Slemrod as a moment approximation to the Boltzmann equation. The relaxed Burnett system is weakly parabolic, has a linearly hyperbolic convection part, and is endowed with a generalized entropy inequality. It agrees with the solution of the Boltzmann equation up to the Burnett order via the Chapman-Enskog expansion.We develop a one-dimensional non-oscillatory numerical scheme based on the relaxed Burnett system for the Boltzmann equation. We compare numerical results for stationary shocks based on this relaxation scheme, and those obained by the DSMC (Direct Simulation Monte Carlo), by the Navier-Stokes equations and by the extended thermodynamics with thirteen moments (the Grad equations). Our numerical experiments show that the relaxed Burnett gives more accurate approximations to the shock profiles of the Boltzmann equation obtained by the DSMC, for a range of Mach numbers for hypersonic flows, than those obtained by the other hydrodynamic systems.

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