On the Combinatorics of the Pfaff Identity

William Y. C. Chen and Sabrina X. M. Pang

Abstract: Recently, there has been a revival of interest in the Pfaff identity on hypergeometric series because of the specialization of Simons and a generalization of Munarini. We present combinatorial settings and interpretations of the specialization and the generalization; one is based on free Dyck paths and free Schröder paths, and the other relies on a correspondence of Foata and Labelle between the Meixner endofunctions and bicolored permutations, and an extension of the technique developed by Labelle and Yeh for the Pfaff identity. Applying the involution on weighted Schröder paths, we derive a formula for the Narayana numbers as an alternating sum of the Catalan numbers.

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