

Faulhaber's Theorem on Power Sums

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Abstract: We observe that the classical Faulhaber's theorem on sums of odd powers also holds for an arbitrary arithmetic progression, namely, the odd power sums of any arithmetic progression $a+b, a+2b, \dots, a+nb$ is a polynomial in $na + n(n+1)b/2$. While this assertion can be deduced from the original Faulhaber's theorem, we give an alternative formula in terms of the Bernoulli polynomials. Moreover, by utilizing the central factorial numbers as in the approach of Knuth, we derive formulas for r -fold sums of powers without resorting to the notion of r -reflexive functions. We also provide formulas for the r -fold alternating sums of powers in terms of Euler polynomials.

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Keywords: Faulhaber's theorem, power sum, alternating sum, r -fold power sum, r -fold alternating power sum, Bernoulli polynomial, Euler polynomial

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