Partitions of Z_n into Arithmetic Progressions

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Abstract: We introduce the notion of arithmetic progression blocks or AP-blocks of Z_n, which can be represented as sequences of the form $(x, x+m, x+2m,..., x+(i-1)m)$ (*mod n*). Then we consider the problem of partitioning Z_n into AP-blocks for a given difference m. We show that subject to a technical condition, the number of partitions of Z_n into m-AP-blocks of a given type is independent of m. When we restrict our attention to blocks of sizes one or two, we are led to a combinatorial interpretation of a formula recently derived by Mansour and Sun as a generalization of the Kaplansky numbers. These numbers have also occurred as the coefficients in Waring's formula for symmetric functions.

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Return