Families of Sets with Intersecting Clusters

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Abstract: A family of k-subsets $A_1, A_2, \ldots, A_d$ on $[n] = \{1, 2, \ldots, n\}$ is called a (d, c)-cluster if the union $A_1 \cup A_2 \cup \ldots \cup A_d$ contains at most ck elements with $c < d$. Let F be a family of k-subsets of an n-element set. We show that for $k \geqslant 2$ and $n \geqslant k + 2$, if every $(k, 2)$-cluster of F is intersecting, then F contains no $(k - 1)$-dimensional simplices. This leads to an affirmative answer to Mubayi's conjecture for $d = k$ based on Chvatal's simplex theorem. We also show that for any d satisfying $3 \leqslant d \leqslant k$ and $n \geqslant \frac{dk}{d-1}$, if every $(d, d+1/2)$-cluster is intersecting, then $|F| \leqslant \binom{n-1}{k-1}$ with equality only when F is a complete star. This result is an extension of both Frankl's theorem and Mubayi's theorem.

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