

# On Stanley's Partition Function

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**Abstract:** Stanley defined a partition function  $t(n)$  as the number of partitions  $\lambda$  of  $n$  such that the number of odd parts of  $\lambda$  is congruent to the number of odd parts of the conjugate partition  $\lambda'$  modulo 4. We show that  $t(n)$  equals the number of partitions of  $n$  with an even number of hooks of even length. We derive a closed-form formula for the generating function for the numbers  $p(n) - t(n)$ . As a consequence, we see that  $t(n)$  has the same parity as the ordinary partition function  $p(n)$  for any  $n$ . A simple combinatorial explanation of this fact is also provided.

**AMS Classification:** 05A17

**Keywords:** partition function, Jacobi's triple product identity, hook length

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