

On Balanced Colorings of the n -Cube

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Abstract: A 2-coloring of the n -cube in the n -dimensional Euclidean space can be considered as an assignment of weights of 1 or 0 to the vertices. Such a colored n -cube is said to be balanced if its center of mass coincides with its geometric center. Let $B_{n,2k}$ be the number of balanced 2-colorings of the n -cube with $2k$ vertices having weight 1. Palmer, Read and Robinson conjectured that for $n \geq 1$, the sequence $\{B_{n,2k}\}_{k=0,1,\dots,2^{n-1}}$ is symmetric and unimodal. We give a proof of this conjecture. We also propose a conjecture on the log-concavity of $B_{n,2k}$ for fixed k , and by probabilistic method we show that it holds when n is sufficiently large.

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