Oscillating Rim Hook Tableaux and Colored Matchings

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Abstract: Motivated by the question of finding a type B analogue of the bijection between oscillating tableaux and matchings, we find a correspondence between oscillating m-rim hook tableaux and m-colored matchings, where m is a positive integer. An oscillating m-rim hook tableau is defined as a sequence $(\lambda^0, \lambda^1,..., \lambda^{2n})$ of Young diagrams starting with the empty shape and ending with the empty shape such that λ^i is obtained from λ^{i-1} by adding an m-rim hook or by deleting an m-rim hook. Our bijection relies on the generalized Schensted algorithm due to White. An oscillating 2-rim hook tableau is also called an oscillating domino tableau. When we restrict our attention to two column oscillating domino tableaux of length $2n$, we are led to a bijection between such tableaux and noncrossing 2-colored matchings on $\{1, 2,..., 2n\}$, which are counted by the product $C_n C_{n+1}$ of two consecutive Catalan numbers. A 2-colored matching is noncrossing if there are no two arcs of the same color that are intersecting. We show that oscillating domino tableaux with at most two columns are in one-to-one correspondence with Dyck path packings. A Dyck path packing of length $2n$ is a pair (D, E), where D is a Dyck path of length $2n$, and E is a dispersed Dyck path of length $2n$ that is weakly covered by D. So we deduce that Dyck path packings of length $2n$ are counted by $C_n C_{n+1}$.

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Return