Euler's Partition Theorem with Upper Bounds on Multiplicities

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Abstract: We show that the number of partitions of n with alternating sum k such that the multiplicity of each part is bounded by $2m + 1$ equals the number of partitions of n with k odd parts such that the multiplicity of each even part is bounded by m. The first proof relies on two formulas with two parameters that are related to the four-parameter formulas of Boulet. We also give a combinatorial proof of this result by using Sylvester's bijection, which implies a stronger partition theorem. For $m = 0$, our result reduces to Bessenrodt's refinement of Euler's partition theorem. If the alternating sum and the number of odd parts are not taken into account, we are led to a generalization of Euler's partition theorem, which can be deduced from a theorem of Andrews on equivalent upper bound sequences of multiplicities. Analogously, we show that the number of partitions of n with alternating sum k such that the multiplicity of each even part is bounded by $2m + 1$ equals the number of partitions of n with k odd parts such that the multiplicity of each even part is also bounded by $2m + 1$. We provide a combinatorial proof as well.

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