

The Sorting Index and Permutation Codes

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Abstract: In the combinatorial study of the coefficients of a bivariate polynomial that generalizes both the length and the reflection length generating functions for finite Coxeter groups, Petersen introduced a new Mahonian statistic sor , called the sorting index. Petersen proved that the pairs of statistics (sor, cyc) and $(inv, rl-min)$ have the same joint distribution over the symmetric group, and asked for a combinatorial proof of this fact. In answer to this question, we observe a connection between the sorting index and the B-code of a permutation defined by Foata and Han, and we show that the bijection of Foata and Han serves the purpose of mapping $(inv, rl-min)$ to (sor, cyc) . We also give a type B analogue of the bijection of Foata and Han, and derive the equidistribution of $(inv_B, Lmap_B, Rmil_B)$ and $(sor_B, Lmap_B, Cyc_B)$ over signed permutations. So we get a combinatorial interpretation of Petersen's equidistribution of $(inv_B, nmin_B)$ and (sor_B, l_B') . Moreover, we show that the six pairs of set-valued statistics $(Cyc_B, Rmil_B)$, $(Cyc_B, Lmap_B)$, $(Rmil_B, Lmap_B)$, $(Lmap_B, Rmil_B)$, $(Lmap_B, Cyc_B)$ and $(Rmil_B, Cyc_B)$ are equidistributed over signed permutations. For Coxeter groups of type D , Petersen showed that the two statistics inv_D and sor_D are equidistributed. We introduce two statistics $nmin_D$ and \tilde{i}_D for elements of D_n and we prove that the two pairs of statistics $(inv_D, nmin_D)$ and (sor_D, \tilde{i}_D) are equidistributed.

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