On 021-Avoiding Ascent Sequences

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Abstract: Ascent sequences were introduced by Bousquet-Mélou, Claesson, Dukes and Kitaev in their study of (2 + 2)-free posets. An ascent sequence of length n is a nonnegative integer sequence $x = x_1 x_2 \dots x_n$ such that $x_1 = 0$ and $x_i \leq \operatorname{asc}(x_1 x_2 \dots x_{i-1}) + 1$ for all $1 < i \leq n$, where $\operatorname{asc}(x_1 x_2 \dots x_{i-1})$ is the number of ascents in the sequence $x_1 x_2 \dots x_{i-1}$. We let A_n stand for the set of such sequences and use $A_n(p)$ for the subset of sequences avoiding a pattern p. Similarly, we let $S_n(\tau)$ be the set of τ -avoiding permutations in the symmetric group S_n . Duncan and Steingrímsson have shown that the ascent statistic has the same distribution over $A_n(021)$ as over $S_n(132)$. Furthermore, they conjectured that the pair (asc, rlm) is equidistributed over $A_n(021)$ and $S_n(132)$ where rlm is the right-to-left minima statistic. We prove this conjecture by constructing a bistatistic-preserving bijection.

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Keywords: 021-avoiding ascent sequence, 132-avoiding permutation, right-to-left minimum, number of ascents, bijection

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