Proof of Moll's Minimum Conjecture

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Abstract: Let $d_i(m)$ denote the coefficients of the Boros-Moll polynomials. Moll's minimum conjecture states that the sequence $\{i(i+1)(d_i^2(m)-d_{i-1}(m)d_{i+1}(m))\}_{1\leqslant i\leqslant m}$ attains its minimum at $i = m$ with $2^{-2m}m(m+1)\binom{2m}{m}^2$. This conjecture is a stronger than the log-concavity conjecture of Moll proved by Kauers and Paule. We give a proof of Moll's conjecture by utilizing the spiral property of the sequence $\{d_i(m)\}_{0\leqslant i\leqslant m}$, and the log-concavity of the sequence $\{i!d_i(m)\}_{0\leqslant i\leqslant m}$.

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