On the number of partitions with designated summands

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Abstract: Andrews, Lewis and Lovejoy introduced the partition function $PD(n)$ as the number of partitions of n with designated summands, where we assume that among parts with equal size, exactly one is designated. They proved that $PD(3n+2)$ is divisible by 3 and showed that the generating function of $PD(3n)$ can be expressed as an infinite product of powers of $(1-q^{2n+1})$ times a function $F(q^2)$. We obtain a Ramanujan type identity which implies the congruence for $PD(3n+2)$. We also find an explicit formula for $F(q^2)$, which leads to a formula for the generating function of $PD(3n)$. A formula for the generating function of $PD(3n + 1)$ is also obtained. Our proofs rely on Chan's identity on Ramanujan's cubic continued fraction and some identities on cubic theta functions. By introducing a rank for the partitions with designated summands, we give a combinatorial interpretation of the congruence for $PD(3n+2)$.

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