

# Linked Partitions and Permutation Tableaux

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**Abstract:** Linked partitions are introduced by Dykema in the study of transforms in free probability theory, whereas permutation tableaux are introduced by Steingrímsson and Williams in the study of totally positive Grassmannian cells. Let  $[n] = \{1, 2, \dots, n\}$ . Let  $L(n, k)$  denote the set of linked partitions of  $[n]$  with  $k$  blocks, let  $P(n, k)$  denote the set of permutations of  $[n]$  with  $k$  descents, and let  $T(n, k)$  denote the set of permutation tableaux of length  $n$  with  $k$  rows. Steingrímsson and Williams found a bijection between the set of permutation tableaux of length  $n$  with  $k$  rows and the set of permutations of  $[n]$  with  $k$  weak excedances. Corteel and Nadeau gave a bijection from the set of permutation tableaux of length  $n$  with  $k$  columns to the set of permutations of  $[n]$  with  $k$  descents. In this paper, we establish a bijection between  $L(n, k)$  and  $P(n, k - 1)$  and a bijection between  $L(n, k)$  and  $T(n, k)$ . Restricting the latter bijection to noncrossing linked partitions, we find that the corresponding permutation tableaux can be characterized by pattern avoidance.

**AMS Classification:** 05A05, 05A19

**Keywords:** linked partition, descent, permutation tableau.

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