q-Hook Length Formulas for Signed Labeled Forests

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Abstract: A signed labeled forest is defined as a (plane) forest labeled by $\{1, 2,..., n\}$ along with minus signs associated to some vertices. Signed labeled forests can be viewed as an extension of signed permutations. We define the inversion number, the flag major index and the R-major index on signed labeled forests. They can be considered as type B analogues of the indices for labeled forests introduced by Björner and Wachs. The flag major index for signed labeled forests is based on the flag major index on signed permutations introduced by Adin and Roichman, whereas the R-major index for signed labeled forests is based on the R-major index that we introduce for signed permutations, which is closely related to the major defied by Reiner. We obtain q-hook length formulas by q-counting signed labelings of a given forest with respect to the above indices, from which we see that these three indices are equidistributed for signed labeled forests. Our formulas for the major indices and the inversion number are type B analogues of the formula due to Björner and Wachs. We also give a type D analogue with respect to the inversion number of even-signed labeled forests.

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