Infinitely Log-monotonic Combinatorial Sequences

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Abstract: We introduce the notion of infinitely log-monotonic sequences. By establishing a connection between completely monotonic functions and infinitely log-monotonic sequences, we show that the sequences of the Bernoulli numbers, the Catalan numbers and the central binomial coefficients are infinitely log-monotonic. In particular, if a sequence $\{a_n\}_{n\geqslant 0}$ is log-monotonic of order two, then it is ratio log-concave in the sense that the sequence $\{a_{n+1}/a_n\}_{n\geqslant 0}$ is log-concave. Furthermore, we prove that if a sequence $\{a_n\}_{n\geqslant k}$ is ratio log-concave, then the sequence $\{\sqrt[n]{a_n}\}_{n\geq k}$ is strictly logconcave subject to a certain initial condition. As consequences, we show that the sequences of the derangement numbers, the Motzkin numbers, the Fine numbers, the central Delannoy numbers, the numbers of tree-like polyhexes and the Domb numbers are ratio log-concave. For the case of the Domb numbers D_n, we confirm a conjecture of Sun on the log-concavity of the sequence $\{\sqrt[n]{D_n}\}_{n\geq 1}$.

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Keywords: logarithmically completely monotonic function, infinitely logmonotonic sequence, ratio log-concave, Riemann zeta function

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