Ordered Partitions Avoiding a Permutation Pattern of Length 3

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Abstract: An ordered partition of $[n]=\{1, 2,..., n\}$ is a partition whose blocks are endowed with a linear order. Let $OP_{n,k}$ be set of ordered partitions of $[n]$ with k blocks and $OP_{n,k}(\sigma)$ be set of ordered partitions in $OP_{n,k}$ that avoid a pattern σ. For any permutation pattern σ of length 3, Godbole, Goyt, Herdan and Pudwell obtained formulas for the number of ordered partitions of $[n]$ with 3 blocks avoiding σ as well as the number of ordered partitions of $[n]$ with $n-1$ blocks avoiding σ. They showed that $|OP_{n,k}(\sigma)|=|OP_{n,k}(123)|$ for any permutation σ of length 3. Moreover, they raised a question concerning the enumeration of $OP_{n,k}(123)$, and conjectured that the number of ordered partitions of $[2n]$ with blocks of size 2 avoiding σ satisfied a second order linear recurrence relation. In answer to the question of Godbole, et al., we establish a connection between $|OP_{n,k}(123)|$ and the number $e_{n,d}$ of 123-avoiding permutations of $[n]$ with d descents. Using the bivariate generating function of $e_{n,d}$ given by Barnabei, Bonetti and Silimbani, we obtain the bivariate generating function of $|OP_{n,k}(123)|$. Meanwhile, we confirm the conjecture of Godbole, et al. by deriving the generating function for the number of 123-avoiding ordered partitions of $[2n]$ with n blocks of size 2.

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