On the Positive Moments of Ranks of Partitions

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Abstract: By introducing k-marked Durfee symbols, Andrews found a combinatorial interpretation of $2k$-th symmetrized moment $\eta_{2k}(n)$ of ranks of partitions of n in terms of $(k + 1)$-marked Durfee symbols of n. In this paper, we consider the k-th symmetrized positive moment $\bar{\eta}_k(n)$ of ranks of partitions of n which is defined as the truncated sum over positive ranks of partitions of n. As combintorial interpretations of $\bar{\eta}_{2k}(n)$ and $\bar{\eta}_{2k-1}(n)$, we show that for given k and i with $1 \leqslant i \leqslant k + 1$, $\bar{\eta}_{2k-1}(n)$ equals the number of $(k + 1)$-marked Durfee symbols of n with the i-th rank being zero and $\bar{\eta}_{2k}(n)$ equals the number of $(k + 1)$-marked Durfee symbols of n with the i-th rank being positive. The interpretations of $\bar{\eta}_{2k-1}(n)$ and $\bar{\eta}_{2k}(n)$ are independent of i, and they imply the interpretation of $\eta_{2k}(n)$ given by Andrews since $\eta_{2k}(n)$ equals $\bar{\eta}_{2k-1}(n)$ plus twice of $\bar{\eta}_{2k}(n)$. Moreover, we obtain the generating functions of $\bar{\eta}_{2k}(n)$ and $\bar{\eta}_{2k-1}(n)$.

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Keywords: rank of a partition, k-marked Durfee symbol, moment of ranks

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