

# Proof of a Conjecture of Hirschhorn and Sellers on Overpartitions

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**Abstract:** Let  $\bar{p}(n)$  denote the number of overpartitions of  $n$ . It was conjectured by Hirschhorn and Sellers that  $\bar{p}(40n + 35) \equiv 0 \pmod{40}$  for  $n \geq 0$ . Employing 2-dissection formulas of theta functions due to Ramanujan, and Hirschhorn and Sellers, we obtain a generating function for  $\bar{p}(40n + 35)$  modulo 5. Using the  $(p, k)$ -parametrization of theta functions given by Alaca, Alaca and Williams, we prove the congruence  $\bar{p}(40n + 35) \equiv 0 \pmod{5}$ . Combining this congruence and the congruence  $\bar{p}(4n + 3) \equiv 0 \pmod{8}$  for  $n \geq 0$  obtained by Hirschhorn and Sellers, and Fortin, Jacob and Mathieu, we confirm the conjecture of Hirschhorn and Sellers.

**AMS Classification:** 11P83, 05A17

**Keywords:** overpartition, congruence, theta function, dissection formula,  $(p, k)$ -parametrization

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