

Congruences of Multipartition Functions Modulo Powers of Primes

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Abstract: Let $p_r(n)$ denote the number of r -component multipartitions of n , and let $S_{\gamma, \lambda}$ be the space spanned by $\eta(24z)^\gamma \varphi(24z)$, where $\eta(z)$ is the Dedekind's eta function and $\varphi(z)$ is a holomorphic modular form in $M_\lambda(SL_2(\mathbb{Z}))$. In this paper, we show that the generating function of $p_r\left(\frac{m^k n + r}{24}\right)$ with respect to n is congruent to a function in the space $S_{\gamma, \lambda}$ modulo m^k . As special cases, this relation leads to many well known congruences including the Ramanujan congruences of $p(n)$ modulo 5, 7, 11 and Gandhi's congruences of $p_2(n)$ modulo 5 and $p_8(n)$ modulo 11. Furthermore, using the invariance property of $S_{\gamma, \lambda}$ under the Hecke operator T_{ℓ^2} , we obtain two classes of congruences pertaining to the m^k -adic property of $p_r(n)$.

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Keywords: modular form, partition, multipartition, Ramanujan-type congruence

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