Combinatorial Proof of the Inversion Formula on the Kazhdan-Lusztig *R*-Polynomials

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Abstract: In this paper, we present a combinatorial proof of the inversion formula on the Kazhdan-Lusztig *R*-polynomials. This problem was raised by Brenti. As a consequence, we obtain a combinatorial interpretation of the equi-distribution property due to Verma stating that any nontrivial interval of a Coxeter group in the Bruhat order has as many elements of even length as elements of odd length. The same argument leads to a combinatorial proof of an extension of Verma's equi-distribution to the parabolic quotients of a Coxeter group obtained by Deodhar. As another application, we derive a refinement of the inversion formula for the symmetric group by restricting the summation to permutations ending with a given element.

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