The spt-Crank for Ordinary Partitions

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Abstract: The spt-function $spt(n)$ was introduced by Andrews as the weighted counting of partitions of n with respect to the number of occurrences of the smallest part. Andrews, Garvan and Liang defined the spt-crank of an S-partition which leads to combinatorial interpretations of the congruences of $spt(n)$ mod 5 and 7. Let $N_S(m,n)$ denote the net number of S-partitions of n with spt-crank m. Andrews, Garvan and Liang showed that $N_S(m,n)$ is nonnegative for all integers m and positive integers n, and they asked the question of finding a combinatorial interpretation of $N_S(m,n)$. In this paper, we introduce the structure of doubly marked partitions and define the spt-crank of a doubly marked partition. We show that $N_S(m,n)$ can be interpreted as the number of doubly marked partitions of n with spt-crank m. Moreover, we establish a bijection between marked partitions of n and doubly marked partitions of n. A marked partition is defined by Andrews, Dyson and Rhoades as a partition with exactly one of the smallest parts marked. They consider it a challenge to find a definition of the spt-crank of a marked partition so that the set of marked partitions of $5n+4$ and $7n+5$ can be divided into five and seven equinumerous classes. The definition of spt-crank for doubly marked partitions and the bijection between the marked partitions and doubly marked partitions leads to a solution to the problem of Andrews, Dyson and Rhoades.

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