Bijections behind the Ramanujan Polynomials

William Y. C. Chen and Victor. J. W. Guo

Abstract: The Ramanujan polynomials were introduced by Ramanujan in his study of power series inversions. In an approach to the Cayley formula on the number of trees, Shor discovers a refined recurrence relation in terms of the number of improper edges, without realizing the connection to the Ramanujan polynomials. On the other hand, Dumont and Ramamonjisoa independently take the grammatical approach to a sequence associated with the Ramanujan polynomials and have reached the same conclusion as Shor's. It was a coincidence for Zeng to realize that the Shor polynomials turn out to be the Ramanujan polynomials through an explicit substitution of parameters. On the other side of the story, Shor also discovers a recursion of Ramanujan polynomials which is equivalent to the Berndt-Evans-Wilson recursion under the substitution of Zeng, and asks for a combinatorial interpretation. The objective of this paper is to present a bijection for the Shor recursion, or and Berndt-Evans-Wilson recursion, answering the question of Shor. Such a bijection also leads to a combinatorial interpretation of the recurrence relation originally given by Ramanujan.

keywords: Ramanujan polynomials, bijection, rooted tree, improper edge.

AMS Classification: 11B83, 05C05.

Download: pdf

Return