# The Homogeneous $q$-Difference Operator

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**Abstract:** We introduce a $q$-differential operator $D_{xy}$ on functions in two variables which turns out to be suitable for dealing with the homogeneous form of the $q$-binomial theorem as studied by Andrews, Goldman and Rota, Roman, Ihrig and Ismail, et al. The homogeneous versions of the $q$-binomial theorem and the Cauchy identity are often useful for their specializations of the two parameters. Using this operator, we derive an equivalent form of the Goldman-Rota binomial identity and show that it is a homogeneous generalization of the $q$-Vandermonde identity. Moreover, the inverse identity of Goldman and Rota also follows from our unified identity. We also obtain the $q$-Leibniz formula for this operator. In the last section, we introduce the homogeneous Rogers-Szegö polynomials and derive their generating function by using the homogeneous $q$-shift operator.

**Keywords:** $q$-binomial theorem, Cauchy polynomials, $q$-Vandermonde identity, homogeneous $q$-difference operator, $q$-Leibniz formula, homogeneous Rogers-Szegö polynomials.

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