

# Matrix Identities on Weighted Partial Motzkin Paths

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**Abstract:** We give a combinatorial interpretation of a matrix identity on Catalan numbers and the sequence  $(1, 4, 4^2, 4^3, \dots)$  which has been derived by Shapiro, Woan and Getu by using Riordan arrays. By giving a bijection between weighted partial Motzkin paths with an elevation line and weighted free Motzkin paths, we find a matrix identity on the number of weighted Motzkin paths and the sequence  $(1, k, k^2, k^3, \dots)$  for  $k \geq 2$ . By extending this argument to partial Motzkin paths with multiple elevation lines, we give a combinatorial proof of an identity recently obtained by Cameron and Nkwanta. A matrix identity on colored Dyck paths is also given, leading to a matrix identity for the sequence  $(1, t^2 + t; (t^2 + t)^2, \dots)$ .

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**Keywords:** Catalan number, Schröder number, Dyck path, Motzkin path, partial Motzkin path, free Motzkin path, weighted Motzkin path, Riordan array

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