The Butterfly Decomposition of Plane Trees

William Y.C. Chen, Nelson Y. Li, and Louis W. Shapiro

Abstract: We introduce the notion of doubly rooted plane trees and give a decomposition of these trees, called the butterfly decomposition which turns out to have many applications. From the butterfly decomposition we obtain a one-to- one correspondence between doubly rooted plane trees and free Dyck paths, which implies a simple derivation of a relation between the Catalan numbers and the central binomial coefficients. We also establish a one-to-one correspondence between leaf-colored doubly rooted plane trees and free Schröder paths. The classical Chung-Feller theorem on free Dyck paths and some generalizations and variations with respect to Dyck paths and Schröder paths with flaws turn out to be immediate consequences of the butterfly decomposition and the preorder traversal of plane trees. We obtain two involutions on free Dyck paths and free Schröder paths, leading to two combinatorial identities. We also use the butterfly decomposition to give a combinatorial treatment of the generating function for the number of chains in plane trees due to Klazar. We further study the average size of chains in plane trees with n edges and show that this number asymptotically tends to $\frac{n+9}{6}$.

AMS Classification: 05A15, 05A19, 05C05.

Keywords: Plane tree, doubly rooted plane tree, chains in plane trees, k-colored plane tree, butterfly decomposition, Dyck path, Schröder path.

Download: Pdf

Return