journal of inequalities in pure and applied mathematics
issn: 1443-5756



Volume 1, Issue 1, Article 10

# Weighted Modular Inequalities for Hardy-Type Operators on Monotone Functions

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**Keywords:** Hardy-type operators, modular inequalities, weights, N-functions, characterizations, Orlicz-Lorentz

**Date Received:** 03/11/99

**Date Accepted:** 31/01/00

**Subject Codes:** 26D15,42B25,26A33,46E30

**Editors:** Bohumir Opic,

**Abstract:** If

$$(Kf)(x) = \int_0^x k(x,y)\, f(y)\, dy,$$

$x > 0$, is a Hardy-type operator defined on the cone of monotone functions, then weight characterizations for which the modular inequality

$$Q^{-1}\left(\int_0^\infty Q[\theta(Kf)]w\right) \le P^{-1}\left(\int_0^\infty P[Cf]v\right)$$

holds, are given for a large class of modular functions $P, Q$. Specifically, these functions need not both be $N$-functions, and the class includes the case where $Q \circ P^{-1}$ is concave. Our results generalize those in [7,24], where the case $Q \circ P^{-1}$ convex, with $P, Q$, $N$-function was studied. Applications involving the Hardy averaging operator, its dual, the Hardy-Littlewood maximal function, and the Hilbert transform are also given.

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