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The Ratio Between the Tail of a Series and its Approximating Integral

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Abstract:

For a strictly positive function $f(x)$, let $S(n) = \sum_{k=n}^{\infty} f(k)$ and $I(x) = \int_x^{\infty} f(t)dt$, assumed convergent. If $f'(x)/f(x)$ is increasing, then $S(n)/I(n)$ is decreasing and $S(n+1)/I(n)$ is increasing. If $f''(x)/f(x)$ is increasing, then $S(n)/I(n-\frac{1}{2})$ is decreasing. Under suitable conditions, analogous results are obtained for the ``continuous tail'' defined by $S(x) = \sum_{n=0}^{\infty} f(x+n)$: these results apply, in particular, to the Hurwitz zeta function.