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The Boundary of Weighted Analytic Centers for Linear Matrix Inequalities

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Abstract: We study the boundary of the region of weighted analytic centers for linear matrix inequality constraints. Let be the convex subset of $\mathbb{R}^n$ defined by q simultaneous linear matrix inequalities (LMIs)

$$A^{(j)}(x) := A_0^{(j)} + \sum_{i=1}^{n} x_i A_i^{(j)} \succ 0, \quad j = 1, 2, \dots, q,$$

where $A_i^{(j)}$ are symmetric matrices and $x \in \mathbb{R}^n$. Given a strictly positive vector $\omega = (\omega_1, \omega_2, \dots, \omega_q)$, the *weighted analytic center* $x_{ac}(\omega)$ is the minimizer of the strictly convex function

$$\phi_\omega(x) := \sum_{j=1}^{q} \omega_j \log \det [A^{(j)}(x)]^{-1}$$

over $\mathcal{R}$. The region of weighted analytic centers, $\mathcal{W}$, is a subset of $\mathcal{R}$. We give several examples for which $\mathcal{W}$ has interesting topological properties. We show that every point on a central path in semidefinite programming is a weighted analytic center.

We introduce the concept of the *frame* of $\mathcal{W}$, which contains the boundary points of $\mathcal{W}$ which are not boundary points of $\mathcal{R}$. The frame has the same dimension as the boundary of $\mathcal{W}$ and is therefore easier to compute than $\mathcal{W}$ itself. Furthermore, we develop a Newton-based algorithm that uses a Monte Carlo technique to compute the frame points of $\mathcal{W}$ as well as the boundary points of $\mathcal{W}$ that are also boundary points of $\mathcal{R}$.

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