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Young's Inequality In Compact Operators - The Case Of Equality

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Abstract: If a and b are compact operators acting on a complex separable Hilbert space, and if $p, q \in (1, \infty)$ satisfy $\frac{1}{p} + \frac{1}{q} = 1$, then there exists a partial isometry u such that the initial space of u is $(\ker(|ab^*|))^{\perp}$ and

$$u|ab^*|u^* \leq \frac{1}{p}|a|^p + \frac{1}{q}|b|^q.$$

Furthermore, if $|ab^*|$ is injective, then the operator u in the inequality above can be taken as a unitary. In this paper, we discuss the case of equality of this Young's inequality, and obtain a characterization for compact normal operators.