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# On the Heisenberg-Pauli-Weyl Inequality

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**Abstract:** In 1927, W. Heisenberg demonstrated the impossibility of specifying simultaneously the position and the momentum of an electron within an atom.The following result named, *Heisenberg inequality*, is not actually due to Heisenberg. In 1928, according to H. Weyl this result is due to W. Pauli.The said inequality states, as follows: Assume that $f:\mathbb{R}\to\mathbb{C}$ is a complex valued function of a random real variable $x$ such that $f\in L^2(\mathbb{R})$. Then the product of the second moment of the random real $x$ for $|f|^2$ and the second moment of the random real $\xi$ for $\left|\hat{f}\right|^2$ is at least $E_{|f|^2}/4\pi$, where $\hat{f}$ is the Fourier transform of $f$, such that $\hat{f}(\xi)=\int_R e^{-2i\pi\xi x}f(x)\,dx$ and $f(x)=\int_R e^{2i\pi\xi x}\hat{f}(\xi)\,d\xi$, $i=\sqrt{-1}$ and $E_{|f|^2}=\int_R |f(x)|^2\,dx$. In this paper we generalize the afore-mentioned result to *the higher moments for* $L^2$ *functions* $f$ and establish the *Heisenberg-Pauli-Weyl inequality*.