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On the Refined Heisenberg-Weyl Type Inequality

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Abstract: The well-known *second moment Heisenberg-Weyl inequality (or uncertainty relation)* states: Assume that $f : \mathbb{R} \to \mathbb{C}$ is a complex valued function of a random real variable x such that $f \in L^2(\mathbb{R})$, where $\mathbb{R} = (-\infty, \infty)$. Then the product of the second moment of the random real x for $|f|^2$ and the second moment of the random real ξ for $\left|\widehat{f}\right|^2$ is at least $E_{\mathbf{R},|f|^2}/4\pi$, where $\widehat{f}$ is the Fourier transform of f, $\widehat{f}(\xi) = \int_{\mathbf{R}} e^{-2i\pi\xi x} f(x)\, dx$ and $\overline{f(x)} = \int_{\mathbf{R}} e^{2i\pi\xi x} \hat{f}(\xi)\, d\xi$, and $E_{\mathbf{R},|f|^2} = \int_{\mathbf{R}} |f(x)|^2\, dx$. This uncertainty relation is well-known in classical quantum mechanics. In 2004, the author generalized the afore-mentioned result to *the higher order moments for* $L^2(\mathbb{R})$ *functions* f. In this paper, a refined form of the generalized Heisenberg-Weyl *type* inequality is established.