



On L'Hospital-Type Rules for Monotonicity

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Keywords: L'Hospital-type rules, Monotonicity, Borwein-Borwein-Rooin ratio, Becker-Stark inequalities, Anderson-Vamanamurthy-Vuorinen inequalities, log-concavity, Maclaurin series, Hyperbolic geometry, Right-angled triangles.

Date Received: 18/05/05

Date Accepted: 14/11/05

Subject Codes: 26A48, 26A51, 26A82, 26D10, 50C10, 53A35

Editors: [Jonathan Borwein](#),

Abstract:

Elsewhere we developed rules for the monotonicity pattern of the ratio $r := f/g$ of two differentiable functions on an interval (a, b) based on the monotonicity pattern of the ratio $\rho := f'/g'$ of the derivatives. Those rules are applicable even more broadly than L'Hospital's rules for limits, since in general we do not require that both f and g , or either of them, tend to 0 or ∞ at an endpoint or any other point of (a, b) . Here new insight into the nature of the rules for monotonicity is provided by a key lemma, which implies that, if ρ is monotonic, then $\bar{\rho} := r' \cdot g^2/|g'|$ is so; hence, r' changes sign at most once. Based on the key lemma, a number of new rules are given. One of them is as follows: Suppose that $f(a+) = g(a+) = 0$; suppose also that $\rho \nearrow \searrow$ on (a, b) - that is, for some $c \in (a, b)$, $\rho \nearrow$ (ρ is increasing) on (a, c) and $\rho \searrow$ on (c, b) . Then $r \nearrow$ or $\nearrow \searrow$ on (a, b) . Various applications and illustrations are given.



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