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On The Sharpened Heisenberg-Weyl Inequality

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Abstract: The well-known *second order moment Heisenberg-Weyl inequality (or uncertainty relation)* in Fourier Analysis states: Assume that $f : \mathbb{R} \to \mathbb{C}$ is a complex valued function of a random real variable x such that $f \in L^2(\mathbb{R})$. Then the product of the second moment of the random real x for $|f|^2$ and the second moment of the random real ξ for $\left|\hat{f}\right|^2$ is at least $E_{|f|^2}/4\pi$, where $\hat{f}$ is the Fourier transform of f, such that $\hat{f}(\xi) = \int_{\mathbf{R}} e^{-2i\pi\xi x} f(x)\, dx$, $f(x) = \int_{\mathbf{R}} e^{2i\pi\xi x} \hat{f}(\xi)\, d\xi$, and $E_{|f|^2} = \int_{\mathbf{R}} |f(x)|^2\, dx$.

This uncertainty relation is well-known in classical quantum mechanics. In 2004, the author generalized the afore-mentioned result to *higher order moments* and in 2005, he investigated a Heisenberg-Weyl *type* inequality *without Fourier transforms.* In this paper, a sharpened form of this generalized Heisenberg-Weyl inequality is established *in Fourier analysis*. Afterwards, an open problem is proposed on some pertinent extremum principle.These results are useful in investigation of quantum mechanics.