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On Rank Subtractivity Between Normal Matrices

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Abstract: The rank subtractivity partial ordering is defined on $\mathbb{C}^{m\times n}$ ($n \geq 2$) by $\mathbf{A} \leq^{-} \mathbf{B} \Leftrightarrow \mathrm{rank}(\mathbf{B}-\mathbf{A}) = \mathrm{rank}\mathbf{B} - \mathrm{rank}\mathbf{A}$, and the star partial ordering by $\mathbf{A} \leq^{*} \mathbf{B} \Leftrightarrow \mathbf{A}^{*}\mathbf{A} = \mathbf{A}^{*}\mathbf{B} \wedge \mathbf{A}\mathbf{A}^{*} = \mathbf{B}\mathbf{A}^{*}$. If $\mathbf{A}$ and $\mathbf{B}$ are normal, we characterize $\mathbf{A} \leq^{-} \mathbf{B}$. We also show that then

$$\mathbf{A} \leq^{-} \mathbf{B} \wedge \mathbf{A}\mathbf{B} = \mathbf{B}\mathbf{A} \Leftrightarrow \mathbf{A} \leq^{*} \mathbf{B} \Leftrightarrow \mathbf{A} \leq^{-} \mathbf{B} \wedge \mathbf{A}^{2} \leq^{-} \mathbf{B}^{2}.$$

Finally, we remark that some of our results follow from well-known results on EP matrices.