# Improved Inclusion-Exclusion Inequalities for Simplex and Orthant Arrangements

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**Abstract:**

Improved inclusion-exclusion inequalities for unions of sets are available wherein terms usually included in the alternating sum formula can be left out. This is the case when a key *abstract tube* condition, can be shown to hold. Since the abstract tube concept was introduced and refined by the authors, several examples have been identified, and key properties of abstract tubes have been described. In particular, associated with an abstract tube is an inclusion-exclusion identity which can be truncated to give an inequality that is guaranteed to be at least as sharp as the inequality obtained by truncating the classical inclusion-exclusion identity.

We present an abstract tube corresponding to an orthant arrangement where the inclusion-exclusion formula terms are obtained from the incidence structure of the boundary of the union of orthants. Thus, the construction of the abstract tube is similar to a construction for Euclidean balls using a Voronoi diagram. However, the proof of the abstract tube property is a bit more subtle and involves consideration of abstract tubes for arrangements of simplicies, and an intricate geometric arguments based on their Voronoi diagrams.

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