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Monotonic Refinements of a Ky Fan Inequality

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Abstract: It is well-known that inequalities between means play a very important role in many branches of mathematics. Please refer to [1,3,7], etc. The main aims of the present article are:

(i)
to show that there are monotonic and continuous functions $H(t)$, $K(t)$, $P(t)$ and $Q(t)$ on $[0,1]$ such that for all $t \in [0,1]$,

$$H_n \leq H(t) \leq G_n \leq K(t) \leq A_n$$

and

$$H_n/(1-H_n) \leq P(t) \leq G_n/G_n' \leq Q(t) \leq A_n/A_n',$$

where A_n, G_n and H_n are respectively the weighted arithmetic, geometric and harmonic means of the positive numbers $x_1, x_2, ..., x_n$ in $(0, 1/2]$, with positive weights $\alpha_1, \alpha_2, ..., \alpha_n$; while A_n' and G_n' are respectively the weighted arithmetic and geometric means of the numbers $1-x_1$, $1-x_2, ..., 1-x_n$ with the same positive weights $\alpha_1, \alpha_2, ..., \alpha_n$;

(ii)
to present more general monotonic refinements for the Ky Fan inequality as well as some inequalities involving means; and

(iii)
to present some generalized and new inequalities in this connection.

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