Volume 2, Issue 3, Article 27

Generalized Auxiliary Problem Principle and Solvability of a Class of Nonlinear Variational Inequalities Involving Cocoercive and Co-Lipschitzian Mappings

Authors: Ram U. Verma,

Keywords: Generalized auxiliary variational inequality problem, Cocoercive mappings, Approximation-solvability, Approximate solutions, Partially relaxed monotone mappings.

Date Received: 31/12/00

Date Accepted: 15/03/01

Subject Codes: 49J40

Editors: Drumi Bainov,

Abstract:

The approximation-solvability of the following class of nonlinear variational inequality (NVI) problems, based on a new generalized auxiliary problem principle, is discussed.

Find an element $x^* \in K$ such that

$$\langle (S-T)(x^*), x - x^* \rangle + f(x) - f(x^*) \geq 0 \text{ for all } x \in K,$$

where $S, T : K \to H$ are mappings from a nonempty closed convex subset K of a real Hilbert space H into H, and $f : K \to \mathbb{R}$ is a continuous convex functional on K. The generalized auxiliary problem principle is described as follows: for given iterate $x^k \in K$ and, for constants $\rho > 0$ and $\sigma > 0$), find x^{k+1} such that

$$\langle \rho(S-T)(y^k) + h'(x^{k+1}) - h'(y^k), x - x^{k+1} \rangle$$
$$+\rho(f(x) - f(x^{k+1})) \geq 0 \text{ for all } x \in K,$$

where

$$\langle \sigma(S-T)(x^k) + h'(y^k) - h'(x^k), x - y^k \rangle$$

$$+\sigma(f(x) - f(y^k)) \geq 0 \quad \text{for all} \quad x \in K,$$

where h is a functional on K and h' the derivative of h.