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Lower Bounds for the Infimum of the Spectrum of the Schrödinger Operator in $\mathbb{R}^n$ and the Sobolev Inequalities

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Abstract:

This article is concerned with the infimum e_1 of the spectrum of the Schrödinger operator $\tau = -\Delta + q$ in $\mathbf{R}^N$, $N \geq 1$. It is assumed that $q_- = \max(0, -q) \in L^p(\mathbf{R}^N)$, where $p \geq 1$ if $N = 1$, $p > N/2$ if $N \geq 2$. The infimum e_1 is estimated in terms of the L^p-norm of q_- and the infimum $\lambda_{N,\theta}$ of a functional $\Lambda_{N,\theta}(v) = \|\nabla v\|_2^\theta \|v\|_2^{1-\theta} \|v\|_r^{-1}$, with v element of the Sobolev space $H^1(\mathbf{R}^N)$, where $\theta = N/(2p)$ and $r = 2N/(N-2\theta)$. The result is optimal. The constant $\lambda_{N,\theta}$ is known explicitly for $N = 1$; for $N \geq 2$, it is estimated by the optimal constant $C_{N,s}$ in the Sobolev inequality, where $s = 2\theta = N/p$. A combination of these results gives an explicit lower bound for the infimum e_1 of the spectrum. The results improve and generalize those of Thirring [*A Course in Mathematical Physics III. Quantum Mechanics of Atoms and Molecules*, Springer, New York 1981] and Rosen [*Phys. Rev. Lett.*, **49** (1982), 1885-1887] who considered the special case $N = 3$. The infimum $\lambda_{N,\theta}$ of the functional $\Lambda_{N,\theta}$ is calculated numerically (for $N = 2, 3, 4, 5,$ and 10) and compared with the lower bounds as found in this article. Also, the results are compared with these by Nasibov [*Soviet. Math. Dokl.*, **40** (1990), 110-115].